Multifactorial Decomposition of Inequality: The Case of CAP[1]

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The purpose of this article is to propose a theoretical foundation on the impact of a transfer scheme on income inequality in the redistribution process among participants in a related agreement. Our example involves the study of the Common Agricultural Policy implemented by EU Countries. First, we show that ex-post inequality (after the distribution process) may increase if either initial aggregate income or the amounts of fiscal contributions are sufficiently high. Second, we characterize a multi-factorial methodology according to Palestini and Pignataro (2014) to gauge the impact of redistribution and the effects of different income sources to the inequality profile. Finally, we propose an exercise where a hypothetical policy is implemented and we apply the Banzhaf and Shapley values to determine the marginal contributions of each factor to overall inequality.

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Introduction

Since the 1970s European countries have systematically employed part of their direct taxation and collections of rents to fund specific activities and to redistribute incomes across countries. The primary objective of these policies is to attract firms in order to decrease regional income inequality, increase employment, wages, and productivity in those regions. It seems, however, that these transfer schemes have been unsuccessful. Political parties of all shades, farmers' unions, environmental campaigners and taxpayer groups criticized these policies for going either the

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wrong way, or not far enough. Empirical evidence even suggests that although income per capita has slightly converged between Member States, it has diverged at the regional level during the same period; (see Combes and Overman, 2004).

Among all these programs, Common Agricultural Policy (CAP, hereafter) is the most expensive scheme in the European Union. It relies on a system of subsidies and support programs in a mix between direct payments to farmers together with price or market supports[2]. In each State, tax payments imposed to different classes of farmers are largely refunded by a revenue subsidy provided per hectare of land. One of the most debatable aspects of this issue consists in assessing the extent to which the CAP-driven subsidies clearly contribute to the reduction of income inequalities in the agricultural sector. Empirical evidence shows that the majority of payments go to the largest businesses. Schmid *et al.* (2006) state that in 2001, on average for 14 EU countries, 80 percent of direct payments went to only 20 percent of businesses. Empirical evidence is ambiguous. On one side, Keeney (2000) demonstrates that introducing direct payments has contributed to an increasingly balanced income distribution of agricultural households. Hansen and Teuber (2011) suggest that CAP transfers help smoothing differences in farmers' revenues in the German State of Hesse. According to Shorrocks (1984*a*, 1984*b*), they show a positive effect of transfers within States associated to a reduction of inequality per capita in the disposable income across regions. On the other side, Allanson (2006*a*, 2006*b*) takes into account the redistributive impact of agricultural policy as the difference between the inequality rate on income with subsidies and the inequality rate after the deduction of subsidies. He outlines that the policy intervention has exacerbated the farmers' inequality in Scotland because of horizontal inequities in the incidence of transfers.

To the best of our knowledge, theoretical foundations devoted to characterizing the dynamics of farm support programs and their impact on income distribution among citizens still remain unexplored. The aim of this article is thus first to model potential outgoing and incoming transfer schemes among the agents involved in such a scheme. In particular, we show an application to EU Countries that reveals how inequality in each State is affected by a standard procedure of aggregation of individual contributions. We study a simple environment where each Member State levies taxes on its citizens, in compliance with its fiscal law, to raise money to contribute to a global (European) fund. The aggregated amount must be subsequently reallocated across States after a negotiation process, such as a collective bargaining agreement. We do not address the issue of assignment of shares among Member States, but we take the related vector of received subsidies as given[3]. Whenever a State receives funds, it carries out a further internal redistribution, thereby assigning a fraction of funds to its citizens entitled to benefit from it. Income profiles are modified by such transfers over time, and this influences the distribution within a State. Our analysis involves the use of Atkinson index (see Atkinson, 1970), which is helpful to compare inequality levels before and after the transfers process. A relevant problem of redistribution policy can thus be addressed: provided that the transfer policy is convenient for a State, *i.e.*, since the aggregate contribution does not exceed the discounted value of its share from the bargaining (defined as *profitability condition*), does its inequality level increase or decrease? Conditioned to it, two kinds of effects seem to show up: a) the impact of subsidies on the income of agricultural households generates lower inequality with respect to the one in the initial income distribution; b) when the poorest types do not contribute to the amount of outgoing transfers, *i.e.*, when their tax rates are zero, a reduction in inequality is easier to accomplish due to the transferring process.

As a second step we try to explain the total income inequality as a combination of total income sources. In particular, we verify the effect of a specific subsidy on the related income profile when different income sources are taken into account. Most of the studies on this issue have focused on Gini and Theil indices, since they have particular features for decomposing inequality by income sources. Shorrocks (1984*a*) proposes one of the pioneering procedures to accomplish these types of decomposition. He proves that it is possible to derive an infinite number of decompositions without further restrictions, *i.e.*, a property which is called *natural* decomposition and is valid for the main inequality indices. According to this procedure, Lerman and Yitzhaki (1985) propose a decomposition based on the covariance formula of the Gini index building on Fei *et al.* (2013).[4] They obtain the impact of the marginal change in a given income source on overall inequality. We believe that the decomposition by income sources is a particularly helpful methodology to assess the marginal contribution of each factor in terms of public policy. According to Palestini and Pignataro (2014), we exploit a multifactorial extension of the Atkinson index discovering the effect of a redistribution among income sources. We may show that if the profitability condition mentioned above is satisfied, then the inequality may decrease even if, for instance, only one income source is involved in the redistribution process. The multifactorial extension also allows for measuring the marginal impact of the income sources to the aggregate inequalities according to some power indices such as the Banzhaf value (Banzhaf, 1965) and the Shapley value (Shapley, 1953; Owen 1995). An analogous approach has been adopted in the literature on inequality assessment, in particular by Shorrocks (1984*a*), Shorrocks (1984*b*), Chantreuil and Trannoy (2013) and Charpentier and Mussard (2011), whereas Pignataro (2010) and Pignataro (2009) propose respectively an application of Shapley value and a decomposition by income sources in the opportunity egalitarian environment. This procedure is finally implemented in a 4-sources scheme, where tax-free rent, net income and incoming transfers contribute to overall inequality.

The remainder of this article is the following: the first section outlines the necessary notation and introduces the setup and the features of the theoretical results achieved by employing the standard Atkinson index. A second section characterizes the multifactorial structure by looking at the effect of policies among income sources. A third section introduces a simple stylized exercise with four sources, by looking at the effect of policy on inequality level and at the marginal contribution assigned to each factor. In the fourth section, we depict some anecdotal evidence on the Common Agricultural Policy in Europe and then concluding remarks follow in a final section.

Model

Consider a country consisting of n types of agents. We characterize a population by the multiset of incomes of each subgroup. The representative income of type i, x_i, is composed by j income sources, *i.e.*, x_{ij}, with $j = 1,...,m$, such that, for all $i = 1,...,n$, we have $x_i = \sum_{j=1}^{m} x_{ij}$. An overall income distribution in the society is therefore represented by $X = \{\mathbf{x}_1,...,\mathbf{x}_i,...,\mathbf{x}_n\} \in \Re_+^{n\times m}$, while a distribution of the $j-$income sources by $\mathbf{x}_j = \{x_{j1},...,x_{ji},...,x_{jn}\} \in \Re_+^n$, where a representative income unit x_{ji} relates to the $j-$component of the income of type i. Let $p = n \times m$ be the total number of income units in our society such that x_h, for all $h = 1,...,p$ denotes a generic unit of income. Let us define further $e = \{\mu,...,\mu,...\mu,\} \in \Re_+^p$ the hypothetical fair distribution where all income units are equal to:

$$\mu = \frac{1}{p}\sum_{h=1}^{p} x_h = \frac{1}{p}\sum_{j=1}^{m}\sum_{i=1}^{n} x_{ji}$$

We may also define $e_j = \{\mu_j,...,\mu_j,...,\mu_j,\} \in \Re_+^n$ as the distribution where each income unit in the distribution of the $j-$income source receives the same average income, *i.e.*, $\mu_j = \frac{\sum_{i=1}^{n} x_{ji}}{n}$.

We now employ the following definitions and operations.

- First, imagine a series of States belonging to the European Union. In each country, inhabitants are ranked by their total income x_i. Since each type is endowed with p distinct amounts of income units, we indicate the income profile of the t-th State with the vector $X_t = (x_{1t},\ldots,x_{pt})$. Every coordinate of X_t can be looked upon as a subgroup of the t-th State's total population. Our hypothesis is that all subgroups are sufficiently homogeneous and do not contain very different individual types. The aggregate income of the t-th State is therefore $X_t = \sum_{h=1}^{p} x_{ht}$. Hence, X_t represents the *ex-ante* income distribution of the $t-$State.

- Second, denote the vector $Y_t = A_t X_t = (y_{1t},\ldots,y_{pt})$ collects the fractions of all incomes which are provided by the t-th State as its outgoing transfer.

All transfers are collected by the European Union into a unique aggregate capital fund which will be shared by the countries after a bargaining game. Therefore we can view such gross capital as the grand coalition value of a cooperative game, *i.e.*:

$$\sum_{t=1}^{T}\sum_{h=1}^{p} y_{th} = v(T),$$

where $T \in 2^T$ is the coalition of all States. The bargaining game takes place and, consequently, the exogenous solution concept determines the allocation of the benefits gained by each State.

- Finally, each State reallocates its share to one or more specific groups of its citizens according to the incoming transfer scheme based on its own local rule. At the end of the procedure, the final income profile of the $t-$State is the vector

$$Z_t = (z_{1t}, \ldots, z_{pt})$$

referring to the *ex-post* income distribution of State t.

The first step consists in evaluating the initial level of inequality of income profiles employing one of the common inequality indexes, *e.g.* the Atkinson one; see Atkinson (1970).

Subsequently, we take into account the structure of outgoing and incoming transfers. In order to characterize the t-th vector of outgoing transfers, we construct a taxation matrix A_t. $A_t \in M_p(\mathbb{R})$ is a block matrix whose k main diagonal blocks are diagonal matrices. Each of them represents the tax rate for a certain group of citizens involving at least one subgroup, whereas all the off-diagonal blocks are zero matrices. Suppose that each main diagonal block is a $p \times p$ matrix and that the k-th block is a diagonal $p_k \times p_k$ matrix, where $p_{t1} + \cdots + p_{tk} = p$. The blocks of A_t can also be interpreted as the k different fiscal types of the t-th population. By this representation, we are partitioning the population into k distinct types in accordance with their incomes' characteristics.

If $c_{tk} \geq 0$ is the tax rate for the k-th group of citizens of the t-th State, we have the following construction for the t-th vector of outgoing transfers:

$$A_tX_t = \left(\begin{array}{cccc|ccc|cccc} c_{t1} & 0 & \cdots & 0 & & & & & & & \\ \cdots & \cdots & \cdots & \cdots & & & & & & & \\ 0 & \cdots & 0 & c_{t1} & & & & & & & \\ \hline & & & & \cdots & \cdots & \cdots & & & & \\ & & & & \cdots & \cdots & \cdots & & & & \\ & & & & \cdots & \cdots & \cdots & & & & \\ \hline & & & & & & & c_{tk} & 0 & \cdots & 0 \\ & & & & & & & \cdots & \cdots & \cdots & \cdots \\ & & & & & & & 0 & \cdots & 0 & c_{tk} \end{array}\right) \begin{pmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_{pt} \end{pmatrix} =$$

$$= \begin{pmatrix} c_{t1}x_{1t} \\ c_{t1}x_{2t} \\ \vdots \\ c_{t1}x_{p_1,t} \\ c_{t2}x_{p_1+1,t} \\ \vdots \\ c_{t2}x_{p_1+p_2,t} \\ \vdots \\ \vdots \\ c_{tk}x_{pt} \end{pmatrix} = \begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ y_{pt} \end{pmatrix} = Y_t,$$

which is the t-th vector of contributions provided by the t-th State after taxation. Note that there can be some low incomes that are not subject to taxation, and in that case the corresponding tax rate is zero (for example, if the last block corresponds to the lowest incomes, $c_{ik} = 0$). The total contribution provided by the t-th State is given by the sum of coordinates, *i.e.*,

$$C_t := \sum_{h=1}^{p_1} c_{t1}x_{ht} + \sum_{h=p_1+1}^{p_1+p_2} c_{t1}x_{ht} + \cdots + \sum_{h=p_1+\cdots+p_{k-1}}^{p_1+\cdots+p_k} c_{tk}x_{ht}, \qquad (1)$$

then the total amount of all countries' transfers is given by $C := \sum_{t=1}^{T} C_t$. Note that C is the aggregate contribution that is going to be allocated among member States according to a collective agreement as a result of a bargaining process. If we define a cooperative game whose solution concept is the allocation of C among players, we can consider C as the worth of the grand coalition of the game, *i.e.*, $v(T) = C$, see Owen (1995). The topic of the power assessment in EU Countries has been extensively studied in recent years, and there is still a lively debate on the most suitable power index to be used, see Barr and Passarelli (2009), Koczy (2012), Lebreton *et al.* (2012). We avoid delving into this issue, however, since it is beyond the scope of our article.

Suppose that, after the bargaining game, the distribution of C among the States is described by the vector $D := (d_1, \dots, d_t, ..., d_T) \in \Re_+^T$. Subsequently, the t-th State implements an incoming transfer procedure. If d_t is the positive amount that the t-th State must allocate, such quantity will have to be assigned to the specific group of inhabitants that benefit from the funding. We are assuming that the involved group is composed by subgroups that had a specific unique tax rate, say c_{tl}, when they contributed to overall outgoing transfer. This means that the l-th block of the matrix A_t corresponds to the only type which receives a fraction of the incoming transfer. Because d_t must be apportioned according to an intrinsic distribution scheme, we can indicate with $\alpha_{tl}(s) \in [0,\ 1)$ the share of d_t which is assigned to the s-th subgroup in the l-th block, for $s = 1, \dots, p_l$. By construction, for all $t = 1, \dots, T$, we will have that $\alpha_{tl}(1) + \dots + \alpha_{tl}(p_l) = 1$.

Furthermore, assume that the incoming transfer takes place at a precise time β_1, and that it is subject to a discount factor (for example, depending on the ongoing inflation rate) $e^{-\delta\beta_1}$. Then, after the bargaining and transfer processes, the final income vector of the t-th State becomes:

$$Z_t = \begin{pmatrix} x_{1t} - c_{t1}x_{1t} \\ \vdots \\ x_{p_1,t} - c_{t1}x_{p_1,t} \\ \vdots \\ x_{p_1+\dots+p_{l-1}+1,t} - c_{tl}x_{p_1+\dots+p_{l-1}+1,t} + \alpha_{tl}(1)d_t e^{-\delta\beta_1} \\ \vdots \\ x_{p_1+\dots+p_l,t} - c_{tl}x_{p_1+\dots+p_l,t} + \alpha_{tl}(p_l)d_t e^{-\delta\beta_1} \\ \vdots \\ \vdots \\ x_{pt} - c_{tk}x_{pt} \end{pmatrix}. \tag{2}$$

The condition of profitability for the i-th State after the transfers process is given by:

$$X_t \leq Z_t \iff C_t \leq d_t e^{-\delta\beta_1}, \tag{3}$$

whose economic meaning is straightforward: the aggregate amount of outgoing transfers must not exceed the discounted value of the share gained in the bargaining game. Note that (3) implies the positivity of the final aggregate income, *i.e.*, the sum of coordinates of (2). We are going to evaluate the inequality level of Z_t by calculating its Atkinson index of inequality and then carry out a comparison between *ex ante* and *ex post* distributions. In its standard form, given an income distribution $X_t = (x_{1t}, \dots, x_{pt})$, the Atkinson index of inequality I_A reads as follows:

$$I_A(X_t) = 1 - \frac{p^{-\frac{\varepsilon}{1-\varepsilon}} \left[\sum_{h=1}^{p} x_{ht}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{h=1}^{p} x_{ht}}, \tag{4}$$

where $\varepsilon \in (0,\ 1)$ is an aversion parameter. In particular, we have that:

$$I_A(Z_t) = 1 - \frac{p^{-\frac{\varepsilon}{1-\varepsilon}} \left[\sum_{h=1}^{p_1+\dots+p_{l-1}} (x_{ht} - y_{ht})^{1-\varepsilon} + \sum_{h=1}^{p_l} \left(x_{p_1+\dots+p_{l-1}+h,t} - y_{p_1+\dots+p_{l-1}+h,t} + \right.\right.}{\sum_{h=1}^{p} (x_{ht} - y_{ht}) + \sum_{h=1}^{p_l} \left(\alpha_{tl}(h) d_t e^{-\delta\beta_1}\right)}$$

$$\frac{\left.\left. + \alpha_{tl}(s) d_t e^{-\delta\beta_1}\right)^{1-\varepsilon} + \sum_{h=p_1+\dots+p_l+1}^{p} (x_{ht} - y_{ht})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{h=1}^{p} (x_{ht} - y_{ht}) + \sum_{h=1}^{p_l} \left(\alpha_{tl}(h) d_t e^{-\delta\beta_1}\right)}.$$

Before delving into the assessment of such comparison, we prove the following Lemma:

Lemma 1 *Given two vectors* $\mathbf{a} = (a_1, \dots, a_p),\ \mathbf{b} = (b_1, \dots, b_p) \in \mathbb{R}_+^p$, *for any pair of integer numbers* r *and* s *such that* $1 \le r < s < p$, *and for any* $\gamma \in (0,\ 1)$, *if* $\sum_{h=1}^{p} a_h \le \sum_{h=1}^{p} b_h$ *and if*

$$\sum_{h=1}^{p} a_h^{\gamma} \le \sum_{h=1}^{r} b_h^{\gamma} + \sum_{h=r+1}^{r+s} b_h^{\gamma} + \sum_{h=r+s+1}^{p} b_h^{\gamma}. \tag{5}$$

then $I_A(\mathbf{a}) - I_A(\mathbf{b}) \ge 0$.

Proof. Let $\varepsilon = 1 - \gamma$ be the aversion parameter. Calculating the difference between the two Atkinson inequality indices of the vectors yields:

$$I_A(\mathbf{a}) - I_A(\mathbf{b}) = -p^{-\frac{1}{\gamma}+1} \left\{ \frac{\left[\sum_{h=1}^{p} a_h^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{h=1}^{p} a_h} - \frac{\left[\sum_{h=1}^{r} b_h^{\gamma} + \sum_{h=r+1}^{r+s} b_h^{\gamma} + \sum_{h=r+s+1}^{p} b_h^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{h=1}^{p} b_h} \right\} =$$

$$= -p^{-\frac{1}{\gamma}+1} \left\{ \left[\frac{\sum_{h=1}^{p} a_h^{\gamma}}{\left(\sum_{h=1}^{p} a_h\right)^{\gamma}}\right]^{\frac{1}{\gamma}} - \left[\frac{\sum_{h=1}^{r} b_h^{\gamma} + \sum_{h=r+1}^{r+s} b_h^{\gamma} + \sum_{h=r+s+1}^{p} b_h^{\gamma}}{\left(\sum_{h=1}^{p} b_h\right)^{\gamma}}\right]^{\frac{1}{\gamma}} \right\},$$

which is positive if and only if

$$\frac{\sum_{h=1}^{p} a_h^{\gamma}}{\left(\sum_{h=1}^{p} a_h\right)^{\gamma}} \le \frac{\sum_{h=1}^{r} b_h^{\gamma} + \sum_{h=r+1}^{r+s} b_h^{\gamma} + \sum_{h=r+s+1}^{p} b_h^{\gamma}}{\left(\sum_{h=1}^{p} b_h\right)^{\gamma}}. \tag{6}$$

Consider the following quantity:

$$\left(\sum_{h=1}^{p} a_h\right)^{\gamma} \left[\sum_{h=1}^{r} b_h^{\gamma} + \sum_{h=r+1}^{r+s} b_h^{\gamma} + \sum_{h=r+s+1}^{p} b_h^{\gamma}\right].$$

The assumption $\sum_{h=1}^{p} a_h \leq \sum_{h=1}^{p} b_h$ implies that

$$\left(\sum_{h=1}^{p} a_h\right)^{\gamma}\left[\sum_{h=1}^{r} b_h^{\gamma}+\sum_{h=r+1}^{r+s} b_h^{\gamma}+\sum_{h=r+s+1}^{p} b_h^{\gamma}\right] \leq$$

$$\leq \left(\sum_{h=1}^{p} b_h\right)^{\gamma}\left[\sum_{h=1}^{r} b_h^{\gamma}+\sum_{h=r+1}^{r+s} b_h^{\gamma}+\sum_{h=r+s+1}^{p} b_h^{\gamma}\right].$$

Hence Equation 6 holds if we have:

$$\sum_{h=1}^{p} a_h^{\gamma} \leq \sum_{h=1}^{r} b_h^{\gamma}+\sum_{h=r+1}^{r+s} b_h^{\gamma}+\sum_{h=r+s+1}^{p} b_h^{\gamma}. \tag{7}$$

□

Lemma 1 leads us to evaluate the difference between $I_A(X_t)$ and $I_A(Z_t)$.

Proposition 1 *Under the profitability condition Equation 3, if*

$$\sum_{h=1}^{p} x_{ht}^{1-\varepsilon} \leq \sum_{h=1}^{p_1+\cdots+p_{l-1}} (x_{ht}-y_{ht})^{1-\varepsilon}+$$

$$+\sum_{h=1}^{p_l}\left(x_{p_1+\cdots+p_{l-1}+h,t}-y_{p_1+\cdots+p_{l-1}+h,t}+\alpha_{tl}(h)d_t e^{-\delta\beta_1}\right)^{1-\varepsilon}$$

$$+\sum_{h=p_1+\cdots+p_l+1}^{p} (x_{ht}-y_{ht})^{1-\varepsilon},$$

then $I_A(X_t) \geq I_A(Z_t)$.

Proof. It suffices to apply Lemma 1 to the vectors X_t and Z_t, using the parameters $r = p_1+\cdots+p_{l-1}$ and $s = p_l$. □

Proposition 1 suggests that if either the initial aggregate income or the fiscal contributions are sufficiently high, then the level of inequality tends to increase after the transfer process. Adopting the same approach, Proposition 2 characterizes the same assessment when the poorest (*i.e.*, endowed with the lowest incomes) types are granted a full tax exemption. We suppose that the tax rate is zero for all subgroups that appear under the l-th block in the matrix A_t. This means that all types, having an income lower than the farmers' income, do not contribute to the outgoing transfers at all. Such assumption can be expressed by making the related tax rates vanish, *i.e.*:

$$c_{t,l+1} = c_{t,l+2} = \cdots = c_{tk} = 0. \tag{8}$$

Proposition 2 *If Equation 8 holds, under the profitability condition (Equation 3), we have that if*

$$\sum_{h=1}^{p} x_{ht}^{1-\varepsilon} \leq \sum_{h=1}^{p_1+\cdots+p_{l-1}} (x_{ht}-y_{ht})^{1-\varepsilon}+$$

$$+\sum_{h=1}^{p_l}\left(x_{p_1+\cdots+p_{l-1}+h,t}-y_{p_1+\cdots+p_{l-1}+h,t}+\alpha_{tl}(h)d_t e^{-\delta\beta_1}\right)^{1-\varepsilon},$$

then $I_A(X_t) \geq I_A(Z_t)$.

Proof. We can repeat the approach adopted in the proof of Lemma 1 and with the notation employed in Proposition 1, adding the hypothesis that $x_{ht} = y_{ht}$ for all $h = p_1+\cdots+p_{l+1},\ldots,p$. □

Note that the sufficient hypothesis in Proposition 2 is more restrictive than the one in Proposition 1. In the case of no-tax area, the policy seems to be more effective due to the reduction of inequality in the income profile. Intuitively, in the ex-post evaluation, after distributing funds, equality mainly improves since at least one type does not contribute to the policy in the first stage, while major contributors cross-subsidize minor ones who finally get the whole amount of money assigned to the State. Improved agricultural production thus could be seen as one of the overall objectives for unequal reduction in the country. We can also stress that in order to stimulate the impacts for which the funds were designed, they not only need continuity in the ranking profile, but also proper structures in terms of profitability condition for the involved State.

A multifactorial structure for policy evaluation

The evaluation of policy cannot be simply related on the *ex-post* distribution of income profiles but a deeper investigation is required. We believe that some information can be captured disentangling the effect of each source; thus for instance, we may understand how income profile performs on the basis of assigned funds by the related policy. A more complex framework must be introduced where each individual gains her income from more than one source. As discussed in the previous section, the generic income, x_{ji}, is the j-th component of the income of type i. Since the population of n types is subject to m distinct income factors, $p = n \times m$ is the number of income units in the $t-th$ country[5]. Each type is endowed with an income vector, and every income vector collects all income sources of the related type i, *i.e.*, $\mathbf{x_i} = (x_{1i}, x_{2i}, \ldots, x_{mi}) \in \Re_{+}^{m}$. Assuming that at least one of the coordinates of each income vector is strictly positive, we call $X = \{\mathbf{x}_1, \ldots, \mathbf{x}_m\} \in \mathbb{R}_{+}^{n\times m}$ the set of income vectors for the referred State.[6] Note that the circumstances where not all individuals are affected by all income factors is taken into account as well, *i.e.*, some income is allowed to be zero. On reasonable ground, if the i-th type is not affected by the j-th factor, we posit $x_{ji} = 0$, requiring that at least one of the incomes of the same factor is strictly positive, *i.e.*, for each type i there exists at least one factor $\mathscr{F}_k$ such that $x_{ik} > 0$.

According to Palestini and Pignataro (2014), we apply a multifactorial version of the traditional Atkinson index. It allows us to construct a framework able to evaluate the inequality level caused by each source. Given an inequality aversion parameter, ε, and denoting $X_{\mathscr{F}_j}$ as the income vector associated to factor F_j, we propose the *multifactorial* version of equally distributed equivalent (*ede*, hereafter) income, identified as the hypothetical level of income that each type should receive in order to keep the society at the same level of social welfare for each j source, stemmed from the actual income units.

$$x_{ej} = \left[\frac{1}{n} \sum_{i=1}^{n} x_{ij}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{9}$$

for all $j = 1, \ldots, m$. Denote $\mu_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij}$ the average income related to the j-th factor, we can simply express the Atkinson inequality measure for j source as $I_A(X_{\mathscr{F}_j}) = 1 - \frac{x_{ej}}{\mu_j}$, and even more generally,

$$\mathscr{I}_A(X_{\mathscr{F}_1,\ldots,\mathscr{F}_k}) = 1 - \frac{1}{k} \sum_{j=1}^{k} \frac{x_{ej}}{\mu_j}$$

for $k = 2, \ldots, m$. A definition in our framework can be provided,

Definition 1 *Given the income distribution $X \in R_+^{n \times m}$ and the factors $\mathscr{F}_1, \ldots, \mathscr{F}_m$, the multi-factorial Atkinson index of inequality $I_A(X_{\mathscr{F}_1,\ldots\mathscr{F}_m})$ is given by:*

$$\mathscr{I}_A(X_{\mathscr{F}_1,\ldots,\mathscr{F}_m}) = 1 - \frac{1}{m} \sum_{j=1}^{m} \frac{x_{ej}}{\mu_j} = 1 - \frac{n^{-\frac{\varepsilon}{1-\varepsilon}}}{m} \sum_{j=1}^{m} \left(\frac{\left[\sum_{i=1}^{n} x_{ij}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} x_{ij}} \right). \tag{10}$$

The aim of this section, hence, is to compare the *ex ante* and *ex post* distributions in a multifactorial perspective. For a given ex-ante income profile $X = \{\mathbf{x}_1, \ldots, \mathbf{x}_m\}$, the multifactorial version of the Atkinson index, $\mathscr{I}_A$, is:

$$\mathscr{I}_A(X) = 1 - \frac{n^{-\frac{\varepsilon}{1-\varepsilon}}}{m} \sum_{j=1}^{m} \left(\frac{\left[\sum_{i=1}^{n} x_{ij}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} x_{ij}} \right) \tag{11}$$

We can derive the same analysis by looking at the ex-post distribution, Z,

$$\begin{aligned}\mathscr{I}_A(Z) &= 1-\frac{n^{-\frac{\varepsilon}{1-\varepsilon}}}{m}\{\sum_{j=1}^{m_1+\cdots+m_{l-1}}\left(\frac{\left[\sum_{i=1}^{n}(x_{ij}-y_{ij})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n}(x_{ij}-y_{ij})}\right)+ \\ &+\left(\frac{\left[\sum_{i=1}^{n}\left(x_{ij}-y_{ij}+\alpha_{ij}(s)d_ie^{-\delta t_1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n}\left(x_{ij}-y_{ij}+\alpha_{ij}(s)d_ie^{-\delta t_1}\right)}\right)+ \\ &+\sum_{j=m_1+\cdots+m_l+1}^{m}\left(\frac{\left[\sum_{i=1}^{n}(x_{ij}-y_{ij})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n}(x_{ij}-y_{ij})}\right)\},\end{aligned} \tag{12}$$

where for simplicity the policy redistribution positively influences only one class of income source, *i.e.*, m_l.[7] The following Lemmas extend the analysis proposed in the previous section in a multifactorial dimension.

Lemma 2 *Given two matrices* $A=(\mathbf{a}_1,\ldots,\mathbf{a}_j,...,\mathbf{a}_m)\in\Re_+^{n\times m}$ *and* $B=(\mathbf{b}_1,\ldots,\mathbf{b_j},...\mathbf{b}_m)\in\Re_+^{n\times m}$*, income profiles are respectively determined as:* $\mathbf{a}_j=(a_{1j},...,a_{ij},...a_{nj})\in\Re_+^n$ *and* $\mathbf{b}_j=(b_{1j},...,b_{ij},...,b_{nj})\in\Re_+^n$*. Therefore, for any pair of integers r and s such that* $1\le r<s<m$ *and* $\gamma\in(0,\ 1)$*, if* $\sum_{i=1}^n a_{ij}\le\sum_{i=1}^n b_{ij}$*,* $\forall j\in(1,...,m)$*, then,*

$$\sum_{j=1}^{m}\tilde{\mathbf{a}}_j\le\sum_{j=1}^{r}\tilde{\mathbf{b}}_j+\sum_{j=r+1}^{r+s}\tilde{\mathbf{b}}_j+\sum_{j=r+s+1}^{m}\tilde{\mathbf{b}}_j \tag{13}$$

such that $\mathscr{I}_A(A)-\mathscr{I}_A(B)\ge 0$.

Proof. Let $\varepsilon=1-\gamma$ be the aversion parameter according to Lemma 1. By taking into account the difference between the multifactorial Atkinson indices for the ex-ante and the ex-post income distributions, we obtain that:

$$\begin{aligned}\mathscr{I}_A(A)-\mathscr{I}_A(B) &= -\frac{n^{-\frac{1}{\gamma}+1}}{m}\left\{\sum_{j=1}^{m}\left(\frac{\left[\sum_{i=1}^{n}a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n}a_{ij}}\right)-\sum_{j=1}^{r}\left(\frac{\left[\sum_{i=1}^{n}(b_{ij})^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n}b_{ij}}\right)+\right. \\ &\left.-\sum_{j=r+1}^{r+s}\left(\frac{\left[\sum_{i=1}^{n}b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n}b_{ij}}\right)-\sum_{j=r+s+1}^{m}\left(\frac{\left[\sum_{i=1}^{n}b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n}b_{ij}}\right)\right\}>0\iff\end{aligned}$$

$$\iff \sum_{j=1}^{m}\left(\frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}}\right) - \sum_{j=1}^{r}\left(\frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}\right) +$$

$$-\sum_{j=r+1}^{r+s}\left(\frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}\right) - \sum_{j=r+s+1}^{m}\left(\frac{\left[\sum_{i=1}^{n} (b_{ij})^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}\right) < 0.$$

For $\forall j \in (1,\ldots,m)$ if $\sum_{i=1}^{n} a_{ij} \leq \sum_{i=1}^{n} b_{ij}$, then for $\gamma \in (0,\ 1)$, by Lemma 1 we can easily derive that:

$$\frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}} \leq \frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}} \tag{14}$$

Aggregating the income profiles among all sources,

$$\sum_{j=1}^{m}\left(\frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}}\right) \leq \sum_{j=1}^{r}\left(\frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}\right) + \sum_{j=r+1}^{r+s}\left(\frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}\right) + \sum_{j=r+s+1}^{m}\left(\frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}\right) \tag{15}$$

Finally, denoting $\tilde{\mathbf{a}}_j$ and $\tilde{\mathbf{b}}_j$ respectively as,

$$\tilde{\mathbf{a}}_j = \frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}} \quad \tilde{\mathbf{b}}_j = \frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}$$

We can simply rewrite Equation 15,

$$\sum_{j=1}^{m} \tilde{\mathbf{a}}_j \leq \sum_{j=1}^{r} \tilde{\mathbf{b}}_j + \sum_{j=r+1}^{r+s} \tilde{\mathbf{b}}_j + \sum_{j=r+s+1}^{m} \tilde{\mathbf{b}}_j, \tag{16}$$

confirming Equation 13 above. Therefore $\mathscr{I}_A(A) - \mathscr{I}_A(B) > 0$.

Lemma 2 depicts a sufficient condition able to guarantee a fair intervention. The ex-ante level of inequality must be lower than the ex-post one because the profitability condition is satisfied for each source. The sufficient condition of profitability among agents for a given source translates directly into a profitable gain for all sources. Interestingly, it suggests that for each $j \in (1,\ldots,m)$, the amount of funds collected through taxation should be lower than the aggregate income redistributed by the intervention. This is a strong assumption and it just provides a different perspective to guarantee meanwhile efficiency and fairness in the redistribution process of European policy.

We can still go a step further observing how some weaker requirements can be provided to obtain a similar effect of redistribution. Suppose in particular that a requirement of profitability is relaxed for some sources, *i.e.*, the ex-ante profile is higher than the ex-post profile, *i.e.*, $\sum_{i=1}^{n} a_{ij} \geq \sum_{i=1}^{n} b_{ij}$, for some $j \in (1,...,m)$. This means that for some income factors (and the corresponding individuals benefiting from those sources) the amount of taxation is higher than the potential redistributive fund. The following Lemma intends to discuss the circumstances where profitability is partial, such that the relation (Equation 3) does not hold at a general level.

Lemma 3 *Given two matrices* $A = (\mathbf{a}_1, \ldots, \mathbf{a}_j, ..., \mathbf{a}_m) \in \Re_+^{n\times m}$ *and* $B = (\mathbf{b}_1, \ldots, \mathbf{b_j}, ...\mathbf{b}_m) \in \Re_+^{n\times m}$, *income profiles are respectively determined as:* $\mathbf{a}_j = (a_{1j}, ..., a_{ij}, ...a_{nj}) \in \Re_+^n$ *and* $\mathbf{b}_j = (b_{1j}, ..., b_{ij}, ..., b_{nj}) \in \Re_+^n$. *Therefore, for any pair of integers r and s such that* $1 \leq r < s < m$ *and* $\gamma \in (0,\ 1)$, *if* $\sum_{i=1}^{n} a_{ij} \leq \sum_{i=1}^{n} b_{ij}$ *for some sources, while* $\sum_{i=1}^{n} a_{ij} \geq \sum_{i=1}^{n} b_{ij}$ *for some other sources, therefore,* $\forall j \in (1,...,m)$, *the profitability condition requires that*

$$\sum_{j=1}^{r} (\tilde{\mathbf{a}}_j - \tilde{\mathbf{b}}_j) + \sum_{j=r+s+1}^{m} (\tilde{\mathbf{a}}_j - \tilde{\mathbf{b}}_j) \leq \sum_{j=r+1}^{r+s} (\tilde{\mathbf{a}}_j - \tilde{\mathbf{b}}_j) \tag{17}$$

then $\mathscr{I}_A(A) - \mathscr{I}_A(B) \geq 0$.

Proof. Let $\varepsilon = 1 - \gamma$ be the aversion parameter according to Lemma 1. Lemma 2 suggests that for $\forall j \in (1,...,m)$, $\sum_{i=1}^{n} a_{ij} \leq \sum_{i=1}^{n} b_{ij}$ is a sufficient condition such that $\sum_{j=1}^{m} \mathbf{a}_j \geq \sum_{j=1}^{r} \mathbf{b}_j + \sum_{j=r+1}^{r+s} \mathbf{b}_j + \sum_{j=r+s+1}^{m} \mathbf{b}_j$. If, however, for some income profiles, $\sum_{i=1}^{n} a_{ij} \geq \sum_{i=1}^{n} b_{ij}$; while, for at least one or some other sources, (the ones benefitting from redistribution) results in $\sum_{i=1}^{n} a_{ij} \leq \sum_{i=1}^{n} b_{ij}$, then Equations 14 and 15 do not hold as a general condition anymore. A step further is required, comparing all benefits and losses in all profiles before and after the redistribution. According to Lemma 2, we show that

$$\sum_{j=1}^{r} \left(\frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}} - \frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}} \right) + \sum_{j=r+s+1}^{m} \left(\frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}} - \frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}} \right)$$

$$\leq \sum_{j=r+1}^{r+s} \left(\frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}} - \frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}} \right),$$

while simplifying

$$\sum_{j=1}^{r} (\tilde{\mathbf{a}}_j - \tilde{\mathbf{b}}_j) + \sum_{j=r+s+1}^{m} (\tilde{\mathbf{a}}_j - \tilde{\mathbf{b}}_j) \leq \sum_{j=r+1}^{r+s} (\tilde{\mathbf{a}}_j - \tilde{\mathbf{b}}_j),$$

where $\tilde{\mathbf{a}}_j = \frac{\left[\sum_{i=1}^{n} a_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} a_{ij}}$ and $\tilde{\mathbf{b}}_j = \frac{\left[\sum_{i=1}^{n} b_{ij}^{\gamma}\right]^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} b_{ij}}$. The profitability condition expressed in Equation 17 is the sufficient one to satisfy eq. 16 proposed by Lemma 2.

Lemma 3 suggests that to obtain an effective redistributive policy, the positive difference between ex-ante and ex-post distributions for individuals and sources that benefit from redistribution should be larger than the one among the remaining sources where the taxation is higher than the final contribution. Equation 17 relies on weaker assumption of profitability for our final purpose. It is not necessary to realize a form of redistribution among individuals involving all sources, instead it is sufficient that the net gains from some sources that benefit from policy should be larger than the potential losses held up by citizens through taxation in the other sources. Note that this mechanism of redistribution is completely independent of the heterogeneity of incomes among individuals. Let us imagine some rich individuals. They may still receive more in terms of European contribution although they are paid a high amount of taxation for some sources. What really matters, however, is that the net value from taxation and subsidy for all types is sufficiently high for some income sources such that Equation 17 holds. This can be incorporated in the following proposition where we gauge exactly the difference between $\mathscr{I}_A(X_j) - \mathscr{I}_A(Z_j)$.

Proposition 3 *Under the profitability condition (Equation 3), if*

$$\left(\frac{\left[\sum_{i=1}^{n} x_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} x_{ij}} - \frac{\left[\sum_{i=1}^{n} \left(x_{ij} - y_{ij} + \alpha_{ij}(s) d_i e^{-\delta t_1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} \left(x_{ij} - y_{ij} + \alpha_{ij}(s) d_i e^{-\delta t_1}\right)} \right) \geq$$

$$\sum_{j=1}^{m_1+\cdots+m_{l-1}} \left(\frac{\left[\sum_{i=1}^{n} x_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} x_{ij}} - \frac{\left[\sum_{i=1}^{n} (x_{ij} - y_{ij})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} (x_{ij} - y_{ij})} \right) +$$

$$+ \sum_{j=m_1+\cdots+m_l+1}^{m} \left(\frac{\left[\sum_{i=1}^{n} x_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} x_{ij}} - \frac{\left[\sum_{i=1}^{n} (x_{ij} - y_{ij})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} (x_{ij} - y_{ij})} \right)$$

then $\mathscr{I}_A(X_j) \geq \mathscr{I}_A(Z_j)$.

Proof. It suffices to apply Lemmas 2 and 3 for $\forall j \in (1, \ldots, m)$ to the matrices X_j and Z_j, using the parameters $r = m_1 + \cdots + m_{l-1}$ and $s = m_l$. □

Proposition 3 verifies the sufficient condition implicitly discussed by Lemma 3. It proposes an analysis by taking into account only one factor, *i.e.*, l , *e.g.*, agricultural income or rent from landing. If the benefits from redistribution are higher than the cost of supporting European policy, then the inequality in the ex-post profile is lower than the inequality in the ex-ante one. This implicitly suggests that whenever funds, collected by each country and cross-examined in the bargaining game, are entirely received by agents through a larger redistribution towards a certain source, then the aggregate level of inequality is reduced.

Multifactorial approach: a simple exercise

Here we are going to enrich the framework outlined above, by describing a transfer scheme analogous to the CAP. First, we compare ex-ante and ex-post distributions by looking at 4 different income sources. Second, we show how to gauge the marginal contribution of inequality related to each income source. This exercise is possible when adopting a cooperative game approach, where the income factors are the players of the game and the inequality index is the characteristic value function. What follows is a brief outline of this procedure. Calling $P = \{\mathscr{F}_1, \ldots, \mathscr{F}_m\}$ the set of income factors, $S \subseteq P$ is a coalition of factors, then for all $j \in S$, there exists at least one income unit, x_{ij}, different from the arithmetic mean $\frac{\sum_{i=1}^{n} x_{ij}}{\mu_j}$.[8] The characteristic function of the game is the inequality function $\mathscr{I}_A(X_S)$ introduced by Definition 1, *i.e.*, the multi-factor Atkinson index evaluated at coalition S, which can be any coalition of factors. $\mathscr{I}_A : 2^{\mathscr{P}} \to R$ associates a real number $I_A(X_S)$ to each coalition S as follows:

$$\mathscr{I}_A(X_S) = 1 - \frac{1}{|S|} \sum_{j \in S} \frac{\widehat{x}_j}{\mu_j} = 1 - \frac{n^{-\frac{\varepsilon}{1-\varepsilon}}}{|S|} \sum_{j \in S} \left(\frac{\left[\sum_{i=1}^{n} x_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n} x_{ij}} \right), \tag{18}$$

where $|S|$ indicates the cardinality for all $S \subseteq P$, $S \neq \emptyset$. In the transfer scheme that we are going to describe, the involved income factors are 4. Some differences occur between the previous approach presented in the first section, "Model", and the present one, due to the multi-factor setup. Specifically, we assume that not all the initial incomes of a State's types are taxed, meaning that a portion of the incomes is tax-free. In this framework, there are k different fiscal types for each State. The following table intends to illustrate sources and types:

The related arithmetic means computed over columns are:

$$\mu_{TR} = \frac{1}{k} \left(\frac{\sum_{i=1}^{n_1} \xi_{i,TR}}{n_1} + \cdots + \frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k} \xi_{i,TR}}{n_k} \right),$$

$$\mu_{GI} = \frac{1}{k} \left(\frac{\sum_{i=1}^{n_1} x_{i,GI}}{n_1} + \cdots + \frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k} x_{i,GI}}{n_k} \right),$$

Tax-free Rent	Gross Income	Outgoing Contribution	Incoming Transfer
$\frac{\sum_{i=1}^{n_1} \xi_{i,TR}}{n_1}$	$\frac{\sum_{i=1}^{n_1} x_{i,GI}}{n_1}$	$\frac{-c_{i1}\sum_{i=1}^{n_1} x_{i,OC}}{n_1}$	0
$\frac{\sum_{i=n_1+1}^{n_1+n_2} \xi_{i,TR}}{n_2}$	$\frac{\sum_{i=1}^{n_1+n_2} x_{i,GI}}{n_2}$	$\frac{-c_{i2}\sum_{i=n_1+1}^{n_1+n_2} x_{i,OC}}{n_2}$	0
...	...	...	0
$\frac{\sum_{i=n_1+\dots+n_{l-1}+1}^{n_1+\dots+n_l} \xi_{i,TR}}{n_l}$	$\frac{\sum_{i=n_1+\dots+n_{l-1}+1}^{n_1+\dots+n_l} x_{i,GI}}{n_l}$	$-\frac{c_{il}\sum_{i=n_1+\dots+n_{l-1}+1}^{n_1+\dots+n_l} x_{i,OC}}{n_l}$	$\frac{\sum_{i=1}^{n_l}\left(\alpha_{il}(i)d_{i,IT}e^{-\delta\beta_1}\right)}{n_l}$
...	...	...	0
$\frac{\sum_{i=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k} \xi_{i,TR}}{n_k}$	$\frac{\sum_{i=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k} x_{i,GI}}{n_k}$	$-\frac{c_{ik}\sum_{h=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k} x_{i,OC}}{n_k}$	0

$$\mu_{OC} = -\frac{1}{k}\left(\frac{c_{i1}\sum_{i=1}^{n_1} x_{i,OC}}{n_1} + \dots + \frac{c_{ik}\sum_{i=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k} x_{i,OC}}{n_k}\right),$$

$$\mu_{IT} = \frac{1}{kn_l}\sum_{i=1}^{n_l}\left(\alpha_{il}(i)d_{i,IT}e^{-\delta t_1}\right).$$

We evaluate the inequality induced by more than a group of sources $\mathscr{F}_1,\dots\mathscr{F}_m$ according to the multi-factorial version of the Atkinson index of inequality $\mathscr{I}_A(X_{\mathscr{F}_1,\dots\mathscr{F}_m})$ previously discussed. The grand coalition $X_{TR,GI,OC,IT}$ represents the circumstances where all factors are to be considered. In particular, we obtain:

$$\mathscr{I}_A(X_{TR,GI,OC,IT}) = 1 - \frac{1}{4}\left(\frac{\hat{x}_{TR}}{\mu_{TR}} + \frac{\hat{x}_{GI}}{\mu_{GI}} + \frac{\hat{x}_{OC}}{\mu_{OC}} + \frac{\hat{x}_{IT}}{\mu_{IT}}\right),$$

where $\hat{x}_{TR}$, $\hat{x}_{GI}$, $\hat{x}_{OC}$ and $\hat{x}_{IT}$ respectively correspond to the multi-factorial *ede* determined in Equation 9, *i.e.*:

$$\hat{x}_{TR} := \frac{1}{k^{\frac{1}{1-\varepsilon}}}\left[\left(\frac{\sum_{i=1}^{n_1}\xi_{i,TR}}{n_1}\right)^{1-\varepsilon} + \dots + \left(\frac{\sum_{i=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k}\xi_{i,TR}}{n_k}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \quad (19)$$

$$\hat{x}_{GI} := \frac{1}{k^{\frac{1}{1-\varepsilon}}}\left[\left(\frac{\sum_{i=1}^{n_1}x_{i,GI}}{n_1}\right)^{1-\varepsilon} + \dots + \left(\frac{\sum_{i=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k}x_{i,GI}}{n_k}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \quad (20)$$

$$\hat{x}_{OC} := \frac{1}{k^{\frac{1}{1-\varepsilon}}}\left[\left(-\frac{c_{i1}\sum_{i=1}^{n_1}x_{i,OC}}{n_1}\right)^{1-\varepsilon} + \dots + \left(-\frac{c_{ik}\sum_{i=n_1+\dots+n_{k-1}+1}^{n_1+\dots+n_k}x_{i,OC}}{n_k}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \quad (21)$$

$$\hat{x}_{IT} := \frac{1}{k^{\frac{1}{1-\varepsilon}}}\left[\left(\sum_{i=1}^{n_l}\left(\alpha_{il}(i)d_{i,IT}e^{-\delta\beta_1}\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{22}$$

Consequently the extended version of the Atkinson index is,

$$\mathscr{I}_A(X_{TR,GI,OC,IT}) = 1 - \frac{k^{-\frac{\varepsilon}{1-\varepsilon}}}{4}\left(\frac{\left[\left(\frac{\sum_{i=1}^{n_1}\xi_{i,TR}}{n_1}\right)^{1-\varepsilon}+\cdots+\left(\frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k}\xi_{i,GI}}{n_k}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\frac{\sum_{i=1}^{n_1}\xi_{i,TR}}{n_1}+\cdots+\frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k}\xi_{i,GI}}{n_k}}+\right.$$

$$+\frac{\left[\left(\frac{\sum_{i=1}^{n_1}x_{i,TR}}{n_1}\right)^{1-\varepsilon}+\cdots+\left(\frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k}x_{i,TR}}{n_k}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\frac{\sum_{i=1}^{n_1}x_{i,TR}}{n_1}+\cdots+\frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k}x_{i,GI}}{n_k}}+\frac{n_l\left[\left(\sum_{i=1}^{n_l}\left(\alpha_{il}(i)d_{i,IT}e^{-\delta\beta_1}\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\sum_{i=1}^{n_l}\left(\alpha_{il}(i)d_{i,IT}e^{-\delta\beta_1}\right)}+$$

$$\left.+\frac{\left[\left(-c_{1i}\frac{\sum_{i=1}^{n_1}x_{i,OC}}{n_1}\right)^{1-\varepsilon}+\cdots+\left(-c_{ik}\frac{\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k}x_{i,OC}}{n_k}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{-c_{i1}\frac{\sum_{i=1}^{n_1}x_{i,OC}}{n_1}-\cdots-\frac{c_{ik}\sum_{i=n_1+\cdots+n_{k-1}+1}^{n_1+\cdots+n_k}x_{i,OC}}{n_k}}\right). \tag{23}$$

The above income factors can be viewed as players of a cooperative game, whose grand coalition occurs when each one of them is taken into account. For this reason, the inequality level $\mathscr{I}_A(X_{TR,GI,OC,IT})$ measures the ex-post inequality after the transfer scheme. Analogously, the inequality level $\mathscr{I}_A(Y_{TR,GI})$ indicates the ex-ante inequality. Note that this is not properly a cooperative game, meaning that some factors must be necessarily associated, like TR and GI, whereas some further assumptions should be set to ensure the feasibility of the model. For example, when OC is in a coalition, there must be IT too, because if a State contributes, it must have an incoming transfer, and so on. We are not going to analyse the game itself since we are specifically interested in the comparison between the ex ante and ex post inequality levels.

The profitability condition (Equation 3) can be restated in this case by substituting the ex-ante profile X with $\widetilde{X} = \sum_{i=1}^{n}(\xi_i + x_i)$. In order to establish a more general effect of the transfer scheme on inequality, we can make the following comparison:

$$\mathscr{I}_A(X_{TR,GI,OC,IT}) - \mathscr{I}_A(X_{TR,GI}) = 1 - \frac{1}{4}\left(\frac{\hat{x}_{TR}}{\mu_{TR}}+\frac{\hat{x}_{GI}}{\mu_{GI}}+\frac{\hat{x}_{OC}}{\mu_{OC}}+\frac{\hat{x}_{IT}}{\mu_{IT}}\right) - 1 + \frac{1}{2}\left(\frac{\hat{x}_{TR}}{\mu_{TR}}+\frac{\hat{x}_{GI}}{\mu_{GI}}\right) =$$

$$= \frac{1}{4}\left(\frac{\hat{x}_{TR}}{\mu_{TR}} + \frac{\hat{x}_{GI}}{\mu_{GI}}\right) - \frac{1}{4}\left(\frac{\hat{x}_{OC}}{\mu_{OC}} + \frac{\hat{x}_{IT}}{\mu_{IT}}\right) > 0 \iff$$

$$\iff \frac{\hat{x}_{TR}}{\mu_{TR}} + \frac{\hat{x}_{GI}}{\mu_{GI}} > \frac{\hat{x}_{OC}}{\mu_{OC}} + \frac{\hat{x}_{IT}}{\mu_{IT}} \iff \mathscr{I}_A(X_{OC,IT}) > \mathscr{I}_A(X_{TR,GI}).$$

The above condition points out a relevant prerogative of this kind of inequality measurement. Broadly speaking, if we compare the difference on inequality terms between ex ante and ex post situations, inequality decreases according to such measurement. Such reduction occurs if and only if the inequality level attained by considering only the transfer scheme (and neglecting the initial conditions) is larger than the level of ex ante inequality. This fact is independent of possible profitability conditions, meaning that a transfer scheme inducing a decrease in inequality can be always found, given any ex ante income distribution across types.

In standard cooperative game theory, a well-known and explanatory technique allows us to establish the marginal contribution of each income factor to the overall inequality, consisting in the computation of appropriate solution concepts. The most relevant and descriptive solution concepts are the Shapley value, introduced by Nobel laureate L.S. Shapley in 1953 (Shapley, 1953), and the Banzhaf value, introduced in 1965 (see Banzhaf, 1965),[9] whose formulations are recalled and applied to our case as follows.

The Shapley value of the game $(\mathscr{I}_A,\ \mathscr{P})$ is a vector $\Phi(\mathscr{I}_A) = (\phi_1(\mathscr{I}_A), \dots, \phi_m(\mathscr{I}_A)) \in \Re^m$ such that:

$$\phi_j(\mathscr{I}_A) = \sum_{S\subseteq\mathscr{P},\ j\in S} \frac{(m-|S|)!(|S|-1)!}{m!}(\mathscr{I}_A(S) - \mathscr{I}_A(S\setminus\{j\})), \tag{24}$$

for all $j = 1, \dots, m$.

The Banzhaf value of the game $(\mathscr{I}_A,\ \mathscr{P})$ is a vector $\beta(\mathscr{I}_A) = (\beta_1(\mathscr{I}_A), \dots, \beta_m(\mathscr{I}_A)) \in \Re^m$ such that:

$$\beta_j(\mathscr{I}_A) = \frac{1}{2^{m-1}} \sum_{S\subseteq\mathscr{P},\ j\in S} (\mathscr{I}_A(S) - \mathscr{I}_A(S\setminus\{j\})), \tag{25}$$

for all $j = 1, \dots, m$.

We are going to exploit both values in order to define the marginal contributions of factors to inequality. In particular the coordinates of the Shapley value are:

$$\phi_{TR}(\mathscr{I}_A) = \frac{1}{4}\left(1 - \frac{25\hat{x}_{TR}}{12\mu_{TR}} + \frac{13\hat{x}_{GI}}{36\mu_{GI}} + \frac{13\hat{x}_{OC}}{36\mu_{OC}} + \frac{13\hat{x}_{IT}}{36\mu_{IT}}\right), \tag{26}$$

$$\phi_{GI}(\mathscr{I}_A) = \frac{1}{4}\left(1 - \frac{25\hat{x}_{GI}}{12\mu_{GI}} + \frac{13\hat{x}_{TR}}{36\mu_{TR}} + \frac{13\hat{x}_{OC}}{36\mu_{OC}} + \frac{13\hat{x}_{IT}}{36\mu_{IT}}\right), \tag{27}$$

$$\phi_{OC}(\mathscr{I}_A) = \frac{1}{4}\left(1 - \frac{25\hat{x}_{OC}}{12\mu_{OC}} + \frac{13\hat{x}_{TR}}{36\mu_{TR}} + \frac{13\hat{x}_{GI}}{36\mu_{GI}} + \frac{13\hat{x}_{IT}}{36\mu_{IT}}\right), \tag{28}$$

$$\phi_{IT}(\mathscr{I}_A) = \frac{1}{4}\left(1 - \frac{25\hat{x}_{IT}}{12\mu_{IT}} + \frac{13\hat{x}_{TR}}{36\mu_{TR}} + \frac{13\hat{x}_{GI}}{36\mu_{GI}} + \frac{13\hat{x}_{OC}}{36\mu_{OC}}\right). \tag{29}$$

It is straightforward to compute the coordinates of the Banzhaf value of the game as well:

$$\beta_{TR}(\mathscr{I}_A) = \frac{1}{8}\left(1 - \frac{13\hat{x}_{TR}}{3\mu_{TR}} + \frac{11\hat{x}_{GI}}{12\mu_{GI}} + \frac{11\hat{x}_{OC}}{12\mu_{OC}} + \frac{11\hat{x}_{IT}}{12\mu_{IT}}\right), \tag{30}$$

$$\beta_{GI}(\mathscr{I}_A) = \frac{1}{8}\left(1 - \frac{13\hat{x}_{GI}}{3\mu_{GI}} + \frac{11\hat{x}_{TR}}{12\mu_{TR}} + \frac{11\hat{x}_{OC}}{12\mu_{OC}} + \frac{11\hat{x}_{IT}}{12\mu_{IT}}\right), \tag{31}$$

$$\beta_{OC}(\mathscr{I}_A) = \frac{1}{8}\left(1 - \frac{13\hat{x}_{OC}}{3\mu_{OC}} + \frac{11\hat{x}_{TR}}{12\mu_{TR}} + \frac{11\hat{x}_{GI}}{12\mu_{GI}} + \frac{11\hat{x}_{IT}}{12\mu_{IT}}\right), \tag{32}$$

$$\beta_{IT}(\mathscr{I}_A) = \frac{1}{8}\left(1 - \frac{13\hat{x}_{IT}}{3\mu_{IT}} + \frac{11\hat{x}_{TR}}{12\mu_{TR}} + \frac{11\hat{x}_{GI}}{12\mu_{GI}} + \frac{11\hat{x}_{OC}}{12\mu_{OC}}\right). \tag{33}$$

The issue concerning the choice of a suitable solution concept in this framework deserves some debate which will be possibly developed in future papers. As is well-known (such results are summarized in Freixas (2013)), the Shapley value satisfies the efficiency axiom, *i.e.*, $\sum_j \phi_j(\mathscr{I}_A) = \mathscr{I}_A(X_{\mathscr{P}})$, whereas the Banzhaf value does not this axiom. This means that $\Phi(\mathscr{I}_A,\, P)$ actually represents a tool for establishing the factors' contributions to the overall inequality, as it was a global cost (of inequality) to be allocated among sources. The Banzhaf value leaves a fraction of the overall inequality out of the distribution among sources. In other words, this means that it exists a structural part of inequality due to endogenous causes (such as heterogeneity of sources), which cannot be attributed to any specific factor but only by the connection among them (through the analysis of coalitions).

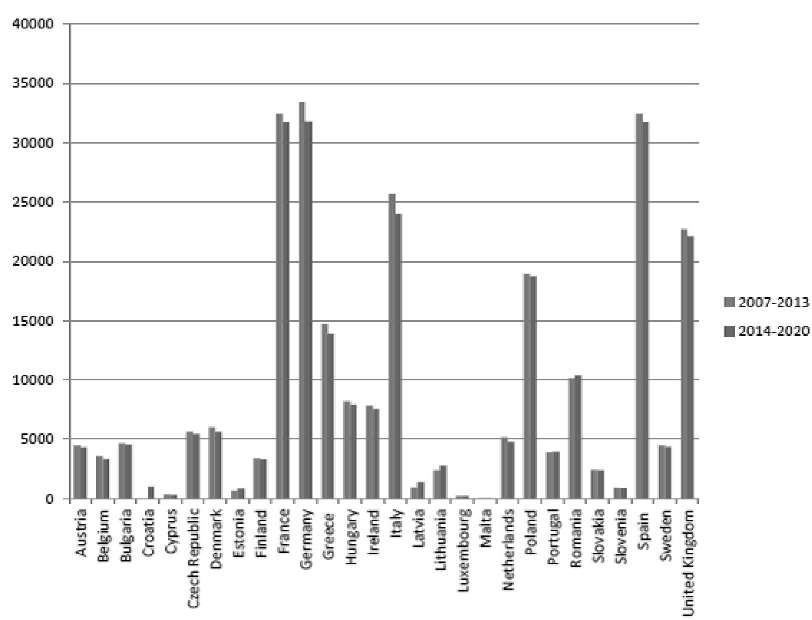
European Common Agricultural Policy

In this section we describe how the transfers' scheme within European Countries contributes to the EU budget and how getting funds from the CAP (direct aid, export refunds and several kinds of subsidies) implies potential redistribution among farmers. The CAP program started in 1962 and was reformed many times. The general idea towards supporting agriculture remains along years in order to ensure a level playing field for farmers competing in the internal and on the global market with a common set of objectives, principles and rules. Without a common policy, Member States would proceed with their own national policies with variable scope and with different degrees of public intervention. Instead the CAP ensures common rules in a single market promoting also a common trade policy since potential re-nationalization of agricultural policy should have a high risk of distorting the common market principle with diverging support levels and types of policy mechanisms. It interchanges credit lines associating some measures funded exclusively by the European Union, and others co-financed by the European Union and national or regional governments. In addition, member states can, to a certain extent, implement purely national measures as long as they follow the basic principles of the European Community for State Aid on agriculture.

The CAP budget nowadays is entirely devoted on income support for farmers, projects in rural development and protection of the environment and support of the market when weather shocks occur (for a simple overview, see Common Agricultural Policy (2012)). A remarkable feature of the CAP is that payments to farmers are in their majority fixed and delivered whether production occurs or not. In 2006 CAP accounted for 46.7% of the total budget, whereas for 2014–2020 such quota is projected to be considerably cut, probably to around 30 percent. The CAP plan has also been affected by wide-ranging criticism (see among others, Gorton (2009) and Matthews (2008)) concerning oversupply, inappropriate environmental development, creation of artificially high food prices and lack of equity among Member States. The CAP costs European taxpayers on average over € 50 billion per year or, in other terms, almost 40 percent of the total EU budget. That is a significant amount of money in particular if we take into account that farming accounts for less than 2 percent of the EU's workforce. Yet the share of the budget devoted to *CAP* spending has fallen sharply: 20 years ago, it was 70 percent. Moreover, the CAP has been reformed significantly, most recently in 2003, when a deal was struck to complete the switch of most CAP subsidies from price supports to direct income payments. After this reform has been fully implemented, some 90 percent of EU farm support will be classified as "non trade-distorting". Perhaps it is helpful to summarize some basic notions on the EU budget (for more details, see European Union, 2002). EU's resources are of three kinds: Traditional Own Resources, consisting of duties that are charged on imports of products coming from non-EU States, which account for 11 percent of the total budget; the resource based on *VAT*, applied to each Member State's own harmonised *VAT* revenue, accounting for approximately 12 percent of the budget; the resource based on GNI, which is the largest source of revenue and accounts for around 76 percent of EU's budget. As can be noted, these three sources are not completely tax-based, whereas our assumptions characterize the transfers of each State to the EU only in terms of fiscal contributions.

Total CAP expenditures are basically addressed into two sources: the first funds come from the European Agricultural Guarantee Fund (EAGF) and accounted for about 80 percent of all CAP EU expenditures, while the second sources come from the European Agricultural Fund for Rural Development (EAFRD). All farmers receiving on average more than €$5,000$ in direct aid have their payments reduced by

8 percent in 2010 (9 percent in 2011 and 10 percent by 2012). A further reduction of 4 percent is made on payments above €300,000 a year. The additional funding obtained this way may be used by member states to reinforce programmes in the fields of climate change, renewable energy, or innovation. In addition, member states may apply additional voluntary modulation, while, the United Kingdom seems to be the only country that currently uses this option. To get a flavour on the policy distribution among countries, we propose in the next figure[10] the contributions to each Member State as direct payments[11], in the periods from 2007 to 2013 and from 2014 to 2020[12].



Note that most Countries achieved a smaller transfer in the second period, due to cuts in EU budget. Intuitively, in the framework proposed above and in order to simplify computations, we consider that each Member State's total contribution to the EU budget is shared among the different types of its population in compliance with the related income profiles, and assuming progressive taxation in all Countries, consequently with the tax rates.

Concluding remarks

The success of European policies in achieving its objectives primarily depends on changes determined by income distributions. We replicate a typical environment of this policy in order to observe its own effect on inequality terms. Our exercise constructs a preliminary characterization of a simple outgoing/incoming transfer scheme to see on one side, the simple effect of a targeted policy on the income profile of a society and on the other side to show how a multi-factorial analysis on this issue may normatively provide different suggestions in terms of welfare gain. To the best of our knowledge, a theoretical setting about the impact of such transfers on inequality distribution of a State is still missing in the on-going literature.

Thus we have first proposed a comparison between the level of inequality before and after the transfer process with the help of the traditional Atkinson index of inequality. We discover that the impact of subsidies on the income of agricultural households generates lower inequality with respect to the one in the initial income distribution. In case of no-tax area, the reduction on inequality is even easier to be accomplished in the ex-post (after the action of transfer) distribution.

Moreover, we believe that a deep understanding about the kind of inequality originated by potential targeted policy can guarantee more interesting evaluation by taking into account each income profile among different sources. This is the reason why a multi-factorial methodology is developed in the second part of the paper in order to capture the impact of each income source and its potential marginal contribution. Indeed, according to Palestini and Pignataro (2014), we implement a multifactorial extension of the Atkinson index. This allows us to understand the effect of redistribution by looking at each source and to discover that a sufficient condition to reduce the inequality simply requires a profitable redistribution. If the beneficial impact of redistribution (even devoted to just one source) is larger than the cost of introducing taxation in the country, then the inequality in the ex-post profile is lower than the inequality in the ex-ante one. We then develop an exercise with four income sources to observe the impact of redistribution and the marginal contribution of the sources through the computation of the Banzhaf value and the Shapley value as alternative solution concepts of a cooperative game on inequality.

Further possible developments may concern more complex formalizations of *CAP* taking into account more factors and improvements of the analysis of the profitability of the transfer scheme. In particular, a clear-cut explanation of the convenience and of the economic consequences of CAP and of other similar schemes would allow to design a more ethical policy.

Appendix

Table 1
Contributions received (or expected) by EU countries between 2007-2020

Country	2007-2013	2014-2020	Country	2007-2013	2014-2020
Austria	4452	4313	Italy	25681	24003
Belgium	3539	3287	Latvia	911	1372
Bulgaria	4652	4547	Lithuania	2363	2744
Croatia	0	1014	Luxembourg	216	209
Cyprus	333	314	Malta	34	31
Czech Republic	5617	5427	Netherlands	5167	4783
Denmark	5999	5642	Poland	18932	18739
Estonia	629	826	Portugal	3897	3940
Finland	3354	3258	Romania	10132	10393
France	32472	31725	Slovakia	2399	2382
Germany	33419	31782	Slovenia	897	856
Greece	14703	13866	Spain	32472	31725
Hungary	8169	7901	Sweden	4463	4337
Ireland	7810	7552	United Kingdom	22705	22148

Notes

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[2]There exist more than 400 kinds of subsidies in Europe. They assume so many forms that it is difficult to classify them as subsidies to production, investment or labor. Some of these funds are granted in the form of interest-rate payments, guarantees and participation in venture capital, while structural funds are mainly proposed as non-repayable grants or indirect aid.

[3]The design of optimal decision rules in the EU has been widely addressed, also in view of its consecutive enlargements. A recent paper by Le Breton *et al.* (2012) provides a complete discussion on the application of several power indices and on the methodology for the assessment of the fair decision rules in the Council of Ministers of the EU across the years, also showing the inadequateness of power among States in some cases.

[4]They show that Gini coefficient for the entire income distribution is equal to the sum of Gini coefficients computed by exploiting the covariance between each income source and the cumulative distribution function of total income.

[5]To simplify notation, the subscript, t, indicating the country is removed in this section hereafter.

[6]In other words, we are implicitly supposing that the factors $\mathscr{F}_1,\ldots\mathscr{F}_m$ are employed to generate X, *i.e.*, there exists a production function $f(\cdot)$ such that $Y = f(\mathscr{F}_1,\ldots\mathscr{F}_m)$.

[7]The same analysis can be extended involving the potential beneficial impacts of redistribution for other sources.

[8]Consequently, when we evaluate inequality related to some factors, we rule out all constant factors, *i.e.*, all elements which assign the same income to all types. Note that in the present setting there are no such factors by construction.

[9]For an exhaustive overview of their properties, axioms, and applications, see Owen (1995).

[10]See the appendix for a detailed description of the amount of money obtained by each country.

[11]Data are expressed in millions of euro.

[12]http://ec.europa.eu/agriculture/rica/pdf/hc0304_distribution_eu25.pdf Data processing: Sole 24 ore.

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